AIAA 79-0059R

Guidance Laws for Short-Range Tactical Missiles

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Introduction

GUIDANCE laws for short-range tactical missiles have become a well-researched topic over the past 35 years with publications of analytical treatment and implementation of missile guidance going back to the 1940's. Thus, much of the guidance development available in the literature predates that which is known as modern control theory. These early concepts, now commonly referred to as classical guidance, have been used from that time to the present to command missiles during their homing phases of flight to target impact. To the extent that assumptions of comparable relative velocity and certain conditions on relative bearing between the pursuer and target were valid, the classical guidance techniques remained adequate. However, evidence is evolving which suggests that the performance of present weapon systems may be seriously degraded in engagements against targets with predicted characteristics of the 1990's and beyond and in the battlefield environments of that time frame. It has been established that the guidance laws currently in wide use may not be adequate in defeating those threats. Thus, it is projected that fundamental advances in the application of control systems theory is required to enhance the guidance effectiveness of future missile weapon systems. Additionally, missile airframe and propulsion systems also may require advances appropriate to defeat predicted targets. In particular, air defense and air-to-air weapons currently undergoing research and development utilizing classical guidance technology may be seriously hampered in the combat scenarios envisioned.

In evaluating the extensive literature on guidance laws applicable to short-range missiles, the lack of a suitable survey became apparent. Although excellent reviews of related topics such as optimal control[1,2] and sensitivity methods[3] had appeared while those fields were relatively young, the missile guidance literature was found to be highly decentralized and fragmented. This may be attributed, in part, to the fact that literature for guidance laws was usually classified for security purposes. These works were rarely published beyond internal industrial reports, and though not made public they became reference documentation for missile designers spanning a period of over 35 years. With classical techniques implemented even in today's highly sophisticated systems, the literature gave credence to a widely accepted definition of classical terminal homing and defined its extensions into the modern era. It was observed that guidance laws typically fit within the following five categories, the first three of which are the well-known classical techniques: line-of-sight (LOS), pursuit, proportional navigation guidance

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Presented as Paper 79-0059 at the AIAA Aerospace Sciences Meeting, New Orleans, La., Jan. 15-17, 1979; submitted Feb. 12, 1979; revision received June 16, 1980.

(PNG), optimal linear, and other guidance laws dominated by differential game methods. The intent of this paper is to highlight briefly the features of each and to provide in one source a short compendium of applicable literature.

In the next section these five categories of guidance laws are described with references to selected articles. Following this discussion, the cost of implementation and the performance characteristics of each category are compared.

The bibliography that follows the paper is by no means exhaustive, but rather representative of the open literature in the guidance area. The terms "guidance" and "control" are often closely associated in this literature. An attempt is made to avoid those references that are basically only control theory oriented. Similarly avoided is the large body of literature on optimization techniques containing many applications to other than terminal homing type problems. Only those references that apply directly to missiles in the context just described are included.

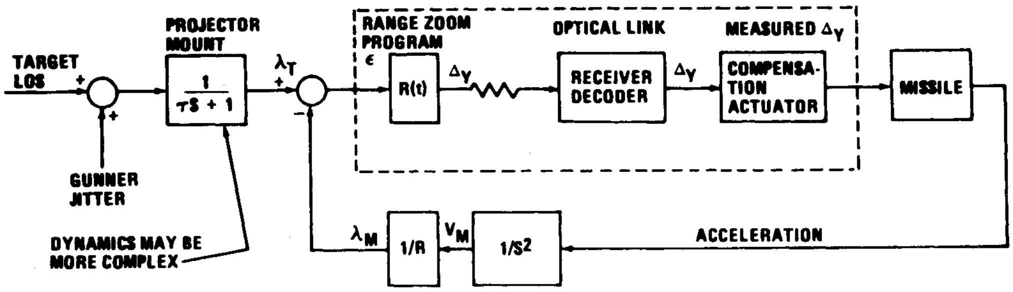
Guidance Laws

In this section, the five categories of guidance schemes previously enumerated are considered, together with their implementation requirements and predicted performance. Several basic texts cover a range of the guidance techniques discussed in this section. Locke[4] and Howe[5] provided detailed developments of LOS, pursuit, and PNG, including actual equations of motion. Teng and Phipps,[6] in a paper motivating nonlinear filtering for missile applications, gave a very readable review of short-range guidance methods, as did Stallard[7] in a tutorial paper on both classical and modern guidance. Basic missile design, including the guidance system, was the thrust of the monograph by Chin.[8]

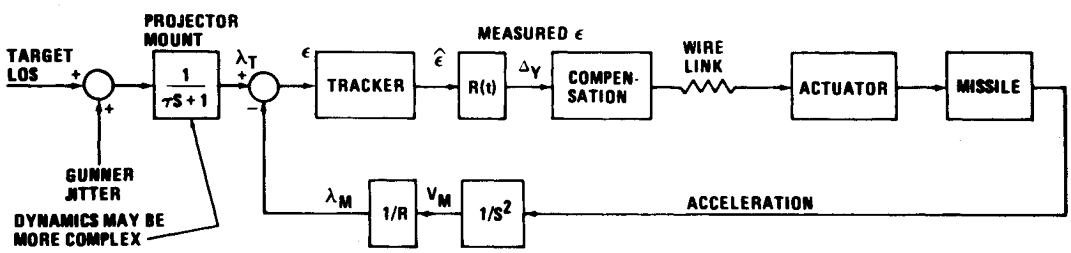
Gregory et al.[9] and later George[10] provided overviews of trends in guidance system design and hardware in the late sixties and midseventies, respectively. Muller[11] described the synthesis of a generic guidance system using inertially stabilized LOS measurements, while Acus[12] presented a comparison of inertial guidance technology applicable to tactical missiles. Two papers by Goodstein[13,14] discussed design tradeoffs and a comparison of the sensitivities of the various guidance laws, respectively. The effects of variations in aerodynamic parameters on missile performance was the focus of a recent paper by Quam.[15] A computer program for missile synthesis was described by Gregory,[16] while statistical analysis programs for missile performance were the subject of reports by Wagner and McAllister,[17] Warren et al.,[18] and Zarchan.[19]

Line-of-Sight Guidance

Line-of-sight (LOS) guidance is often divided into two subsets that differ primarily in their mechanization. Command to line-of-sight (CLOS) guidance typically employs an uplink to transmit guidance signals from a ground controller to the missile. In a beamrider (BR) guidance scheme, the missile attempts to follow an electro-optical beam directed at the target.



a)



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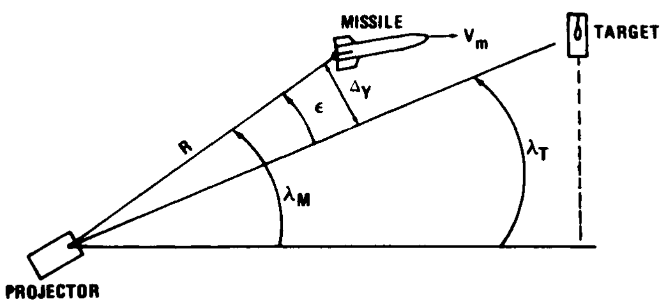


c)

Fig. 1 Line-of-sight guidance: a) beam rider scheme, b) command to line-of-sight scheme, c) line-of-sight geometry.

Clemow[20] provided a detailed development of BR and CLOS implementation in his book. A paper by Harmon et al.[21] described a bang-bang approach to LOS guidance commands, while a pulse duration modulation scheme was explored for a wire-guided missile by Thibodeau and Sharp.[22] Ivanov[23] considered radar based guidance methods and typical implementations of missile borne seekers. Kain and Yost[24] employed a CLOS guidance scheme in ship defense scenario, using optimal linear filters to reduce the inherent beam jitter.

The LOS guidance scheme is one in which the missile is guided on an LOS course in an attempt to remain on a line joining the target and the point of control. To do so, the missile requires a velocity component ($V_{M\perp}$) that is equal to the LOS velocity and perpendicular to it, i.e.,

$$V_{M\perp} = R_{SM}\dot{\lambda}_{ST} \tag{1}$$

where R_{SM} represents the range from the missile to the tracking station and $\dot{\lambda}_{ST}$ represents the LOS rate. In general, the missile flies a pursuit guidance course at the initiation of the entry into the beam at launch and flies an approximately constant bearing course near impact. This is observed from the velocity equation

$$V_{M\perp} = (R_{SM}/R_{ST})V_T \tag{2}$$

where the range of the tracking station to the target is given by R_{ST} and V_T represents the target velocity relative to the surface of the Earth.

Figure 1 contains a simplified control block diagram highlighting features of the scheme. It should be noted that the projector mount dynamics used for the BR and the tracker mount dynamics used for CLOS may be considerably more complex than depicted here. Also, the BR missile requires onboard autopilot compensation since the projector does not know the missile's location once it is enroute. The CLOS scheme, however, does track the missile and thus compensates for its position prior to transmitting the guidance signal via the wire link.

Performance of missiles flying this guidance law is typically very good. Without the man-in-the-loop tracking error, near flawless guidance has been the rule. In the more realistic condition where improper tracking is the major error source, given that a reliable round has been fired, the performance in terms of miss distance has been found to be better than 1 ft with a 90% confidence level.

Pursuit Guidance

One of the most straightforward means to assure an intercept is to keep the missile, which must have velocity superiority, pointed at the target. This is the principle of pursuit guidance, which has two basic variations: attitude pursuit, in which the missile's longitudinal axis is directed at the target; and velocity pursuit, in which the missile's velocity vector is kept pointed at the target.

Derivations of pursuit guidance are provided in the texts by Locke[4] and Howe,[5] and in the review by Teng and Phipps.[6] Papers by Rishel[25] and Goodstein[14] compare the performance and sensitivity of pursuit guidance laws with several other standard guidance techniques.

Both attitude pursuit guidance and velocity pursuit guidance are discussed in the sequel. Attitude pursuit guidance tries to keep the centerline of the missile pointed at the target. In a missile that flies an angle of attack when maneuvering, the velocity vector will always lag the vehicle pointing direction. Miss distance is a strong function of the maneuver capability of the missile and can be reduced by a fast responding high-*g* vehicle.

Velocity pursuit guidance attempts to keep the velocity vector of the missile pointed at the target. In its least sophisticated form, it is mechanized by mounting a target sensor on an air vane that indicates relative wind direction. The difference between this velocity vector and a true velocity vector is the primary error in the scheme. This pursuit guidance mechanization decouples the angle of attack from the target seeker and improves miss distance performance by an amount proportional to the vehicle angle of attack. Figure 2 depicts two-dimensional geometry useful for describing the pursuit guidance laws.

Neglecting the case of a maneuvering target, for simplicity, one notes that the relative closing velocity between missile and target ($\dot{R}$), given in terms of the velocity components along the LOS of the target (V_T) and the missile (V_M), is

$$\dot{R} = V_T\cos(\lambda - \theta_T) - V_M\cos(\lambda - \theta_M) \tag{3}$$

and the LOS rate of rotation is

$$\dot{\lambda} = (V_T - V_M)/R \tag{4}$$

For an ideal pursuit, $\theta_M = \lambda$. Since $\dot{\theta}_M = \dot{\lambda}$, the missile will always have to turn during the attack except for the case of a perfect head or tail chase.

Figure 2 presents an example of a simplified control system diagram for both attitude and velocity pursuit laws. In the former, a wide angle target sensor is required since it is typically mechanized to be body fixed. In the latter, a narrower field-of-view (FOV) sensor may be utilized since the body is decoupled from the sensor mount in a manner previously described. In each case, K_N is the forward guidance gain, and it will differ for each as will the feedback damping gain K_R. The guidance filter in Fig. 2 indicates that higher guidance gain in the velocity pursuit law requires some smoothing to inhibit noise of the target sensor optics and its associated electronics.

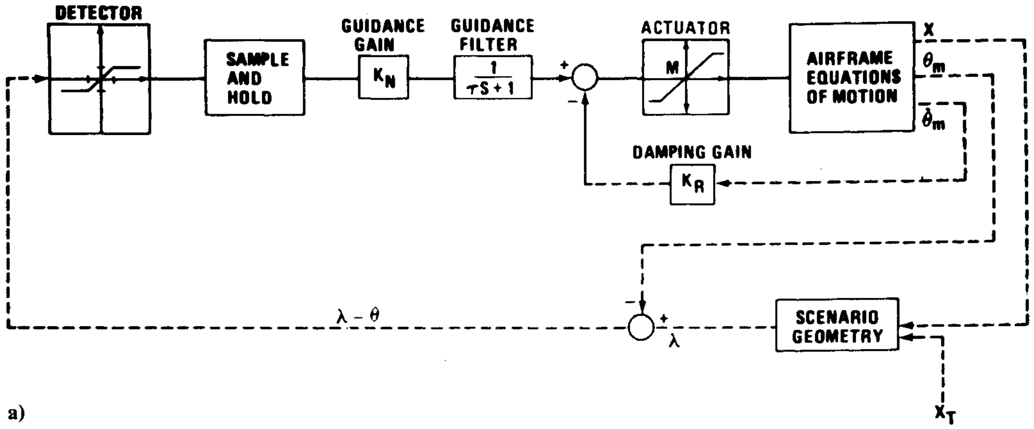
Performance that may be expected from pursuit guidance is indicated in Fig. 3. The data were obtained for a tactical weapon of the class known as close support antitank. Although these are simulation results, experience with flight hardware over the past several years has validated the simulated performance to a high degree of confidence.

Proportional Navigation

Proportional navigation guidance (PNG) probably had its origins among the ancient seafarers who realized that a collision was ensured if two constant velocity vessels maintained a constant relative bearing while closing in range. Thus, unlike pursuit schemes which seek to null the LOS, PNG seeks to null the LOS rate, while closing on the target.

An early paper on guided missile kinematics including the three classic guidance cases of LOS, pursuit, and PNG was written by Newell[26] as a classified monograph during World War II. Its subsequent declassification made it available to the general public. Also, Spits[27] derived the kinematic equations for a PNG guided missile in that same form. This and works by other authors on PNG are summarized in a recent handbook by Paarman et al.[28] Adler[29] considered PNG in three dimensions, and provided a readable development of the theory using vector calculus, whereas most subsequent authors have considered only the planar case for simplification. Irish[30] proposed a PNG scheme for a terminal rendezvous problem, while McElhoe,[31] in a related work, showed that PNG can be used to effect a minimum fuel intercept. In his discussion of general guidance concepts, Wong[32] gave a good overview of PNG. Murtaugh and Criel[33] presented another three-dimensional development in a satellite rendezvous application.

Meyer and Bland[34] considered a variation of PNG in which an on-off sensor is used to provide pulsed guidance commands. Abzug[35] contrasted PNG with a scheme that attempted to null the final projected position error at each guidance update. PNG with a bias term to account for target accelerations normal to the LOS was studied by Brainin and



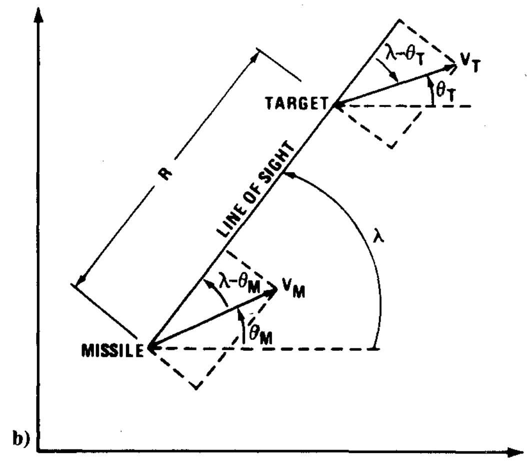


Fig. 2 Pursuit guidance: a) block diagram of one axis—attitude and velocity pursuit guidance, b) pursuit guidance geometry.

McGhee.[36] In their formulation, the magnitude of the target lateral acceleration was available to the interceptor. They showed that a higher effective navigation constant reduced the need for a bias. Khun[37] studied a concept he defined as proportional lead navigation in which the seeker tracked the target with a time constant ensuring a lag.

In a series of papers by Axelband and Hardy,[38-40] a generalization of PNG was obtained by solving a planar quadratic optimization problem. The implementation of the "quasi-optimum PNG" law requires knowledge of the time of intercept, and as shown in the papers, the guidance law is sensitive to the estimate of this quantity. Rawling[41] considered the effects of saturated aerodynamic surfaces on trajectories flown via PNG and concluded that such trajectories would initially have a semicircular segment followed by a constant bearing terminal phase.

Many authors have considered augmenting PNG to account for deviations from the constant velocity assumptions inherent in the derivation. Arbenz[42] suggested making the closing velocity heading rate proportional to the LOS rate and developed a closed-form expression for a modified PNG law. An estimate of target acceleration was added to the missile acceleration command to yield an augmented PNG law in a paper by Siouris.[43]

Guelman[44] used geometric arguments to show the structure of missile trajectories under PNG, proving that for constant velocity targets, intercept would almost always occur. Guelman[45] later extended this work to the case of constant acceleration maneuvering targets. Qualitative trajectories were determined, and target acquisition boundaries were assessed. In another paper, Guelman[46] contrasted "pure" PNG (wherein commanded accelerations are normal to missile velocity) and "true" PNG (wherein commanded accelerations are normal to the LOS). He concluded that the latter law would result in intercept only if the initial conditions were within a well-defined subset of the parameter space.

Pitman[47] compiled sensitivity functions and projected errors for PNG laws with navigation ratio gains of 2, 3, or 4. The effects of ignoring the dynamics of a rolling missile in a PNG terminal homing missile were investigated by Shinar[48] who found that the effective guidance gain was reduced, leading to divergence. Slater and Wells[49] considered a PNG law with a delay and devised optimum evasive tactics by solving a simple quadratic optimization problem. Abzug[50] proposed a PNG algorithm where relative missile target measurements were in inertial coordinates. Nesline[51] evaluated the relationship of signal errors on PNG miss distance and concluded the missile would need a fivefold acceleration superiority over the target to assure a high probability of intercept.

PNG is a guidance law in which the angular rate of the missile flight path is directly proportional to the angular LOS rate of change, i.e.,

$$\dot{\gamma}_M = N\dot{\lambda} \tag{5}$$

where $\dot{\gamma}_M$ represents flight path angular rate relative to a fixed reference, $\dot{\lambda}$ represents the LOS rate relative to a fixed reference, and N is the navigation ratio gain. This is shown simplistically in Fig. 4 for a two-dimensional case. A general expression for missile acceleration may be written

$$\bar{\eta} = V_M \dot{\gamma}_M \bar{V}_{M\perp} + \dot{V}_M \bar{T} \tag{6}$$

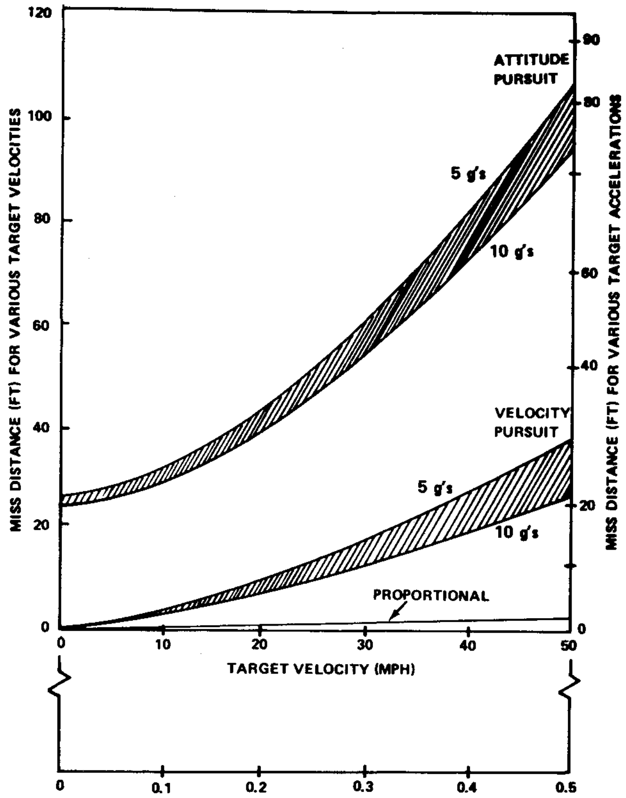


Fig. 3 Performance of classical guidance laws: miss distance vs target velocity/acceleration (control authority indicated in g's).

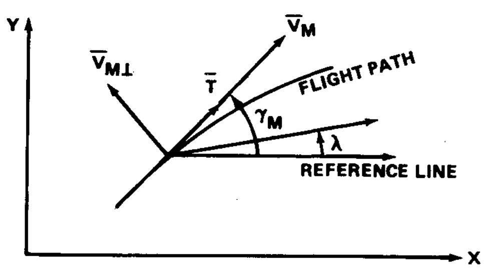


Fig. 4 Geometry of flight path.

where $\overline{\eta}$ represents total missile acceleration, V_M the missile speed, $\dot{\gamma}_M$ the missile flight path angular rate, $\overline{V}_{M\perp}$ a unit vector lateral to the missile flight path, and $\overline{T}$ a unit vector tangential to missile flight path. The definitions of $\overline{V}_{M\perp}$ and $\overline{T}$ yield the relationship, $\overline{V}_{M\perp}\cdot\overline{T}=0$. One may implement proportional navigation via the relationship (in the Laplace domain)

$$\eta_M(s)=k\epsilon(s)=[k\tau s\lambda(s)]/(1+\tau s) \tag{7}$$

where η_M represents the lateral acceleration in g's and ϵ represents the LOS error that is measured by a seeker having a time lag constant of τ and an error of $\dot{\lambda}$. k represents a guidance gain factor. By noting that a constant bearing course is determined if missile and target are flying constant speed and neither is maneuvering, it may be concluded that the LOS at each instant of time would be parallel to the LOS at a previous instant.

The performance and implementation of the guidance law are best appreciated in their relationship to two other guidance laws that are similar in that no requirement is established for range or range rate information. Velocity pursuit and attitude pursuit are in this category with PNG. The performance is indicated in Fig. 3.

Optimal Linear Guidance

Since the mid-1960's the missile guidance literature has become increasingly permeated by techniques based upon optimal control. The great success found by linear-quadratic regulator theory and its dual analog, Kalman filtering, plus the attractive and easily determined form of a feedback solution has led to almost all work in this area being based upon linear model dynamics, with quadratic costs and additive Gaussian noise. This is commonly known as the LQG problem.

Bryson et al.,[52] Denham and Bryson,[53] and Denham,[54] were among the first to apply optimization techniques to missile guidance problems. In a series of papers, they formulated and solved an optimal control problem with inequality constraints and then applied the result to the trajectory shaping of a surface-to-surface missile for range maximization. An overview of the role of optimal control in aerospace applications were provided by Bryson[55] in his Minta Martin Memorial Lecture.

Potter[56] proved separation theory for continuous time guidance and navigation under the assumptions of linear dynamics and measurements, extending a result proved for discrete time models. The use of impulsive velocity corrections with linearized dynamic models and terminal constraints was the subject of papers by Tung[57] and Templeman.[58] McAllister and Schiring[59] applied optimal control theory to thrust vector control in deriving several laws, together with the sensitivity considerations.

In a more theoretical paper, Lee[60] considered the effects of conjugate points on general trajectory optimization problems and showed that the occurrence of conjugate points inhibited the construction of a linear guidance law in a neighborhood of nominal trajectory. Hu and Thompson[61] derived polynomial type guidance functions by considering a general optimization problem, while Talkin[62] developed a variation of PNG using a steepest descent algorithm to minimize the inertial LOS rate.

Both Rang[63] and Rishel[25] considered linear missile models and minimized a quadratic form to achieve a guidance law using state feedback. Rishel incorporated Gaussian noise in his model, used a Kalman filter to generate state estimates, and compared his resulting guidance law with pursuit and PNG schemes. Bashein and Neuman[64] expanded on the stochastic nature of the problem and obtained a linear feedback guidance law that explicitly accounted for model uncertainties. At nearly the same time, Cunningham[65] used a linearized perturbation model about a nominal trajectory to obtain a feedback guidance law.

Another linear-quadratic optimal control formulation was presented by Dickson and Garber,[66] who modeled missile dynamics by a second-order inhomogeneous differential equation. Willems[67] gave solutions to the optimal controller for a missile with antipilot lag defined by two discrete time constants. Andrus et al.[68] compared terminal guidance methods requiring precomputed reference trajectories. Their general approach involved the expansion of end constraints into Taylor series about nominal values. Axelband and Hardy[38,39] used linear optimal control to develop an extension of PNG. However, as with almost all linear schemes, time-to-go was required for implementation.

Stallard[7] gave a good tutorial review of classical and modern methods for homing interceptor missiles. In a later paper, Stallard[69] considered the use of discrete optimal control in a missile system with undesirable stability characteristics. Applying linear optimal guidance to a time-varying stochastic intercept problem, Athans[70] examined the case of optimal resource allocation for simultaneously controlled multiple interceptors and multiple targets. With a change of terminal constraints, Athans' objective may be changed from intercept to rendezvous, allowing for application to a diversity of areas outside short-range missile guidance.

Several authors have used optimal linear guidance to help constrain the attitude of a missile at impact, as well as to help assure target interception. Kim and Grider[71] and Grider et al.[72] reported on simulations wherein a terminal cost was associated with missile attitude. With linear missile and low-order autopilot models, large control gains were observed to be sensitive to the time-to-go. In a later work, York and Pastrick[73] quantified error contributions due to autopilot lag upon target miss distance and attitude angle at impact.

Deyst and Price[74] examined several linear optimal stochastic guidance formulations in planar engagements. Their models included target acceleration as a Markov process and took into account the saturation of control surfaces. The incorporation of such saturations into their problem formulation yielded better results than when the constraints were imposed indirectly via penalty functions.

Asher and Matuzewski[75,76] considered an optimal linear guidance problem where target acceleration was modeled as an external disturbance. They constrained the final miss distance to be zero but, to achieve this, the precise target acceleration history was required. Balbirnie et al.[77] presented a comprehensive study of both classical and modern control techniques for missile guidance, in which a procedure was given to obtain weighting matrices resulting in classical type control gains. In an extension of that work, Sheporaitis et al.[78] examined the effect of placing a quadratic penalty on angle of attack.

Speyer[79] developed a variation of the standard linear optimum solution by minimizing the expected value of a cost having the form $\mu\exp(\frac{1}{2}\mu J)$ where μ is a prespecified scalar and J an integral quadratic form. Speyer's guidance law was obtained in feedback form, but did not enjoy the separation principle; the control gains depended upon the variance in the state estimates. That linear-exponential guidance scheme had the effect of placing very heavy weighting upon large excursions and thus reduced the tails of the terminal miss distribution.

Nazaroff[80] formulated an LQG approach in which he assumed extremely simplified missile dynamics but included target acceleration and jerk terms. Stockum and Weimer[81] assumed exponentially correlated target accelerations in generating an LQG guidance law. The feedback law they obtained was analogous to time-varying PNG. Fiske[82] developed a number of guidance and estimation schemes based upon LQG formulations with varying degrees of model complexity. The closed-form solutions for the controllers were given in his report, allowing the reader to examine the effects of model parameters on the gains. In a similarly oriented tutorial, Gonzalez[83] reviewed LQG formulations of guidance laws for air-to-air missiles. Wei and Pearson[84] examined a planar intercept system in which closed-form solutions to target velocity estimation and minimum energy control were presented.

Most formulations consider terminal miss distance and running control effort only in the cost functional. Unlike the standard regulator format, a running cost on the state is generally not appropriate in this framework, since the problem is to minimize at final time and not continuously during flight. The general optimization problem then becomes cast in the form,

$$\min_{u(\cdot)} \mathrm{E}\{J\} = \mathrm{E}\left\{x'(t_f)S_f x(t_f) + \int_{t_0}^{t_f} u'(t)Ru(t)\,\mathrm{d}t\right\} \tag{8}$$

subject to

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \tag{9}$$

where $\mathrm{E}(\cdot)$ designates the expected value operation, S_f and R weighting matrices, x the state, u the control, and w a white noise process. In most formulations the dynamics are assumed constant to obtain closed-form solutions.

For completeness, the following example summarizes a typical optimal control law formulation. The geometry of the tactical missile-target position is given in Fig. 5. Assume that the angle of attack is small and thus can be neglected (this assumption will be considered later), and choose the following set of variables:

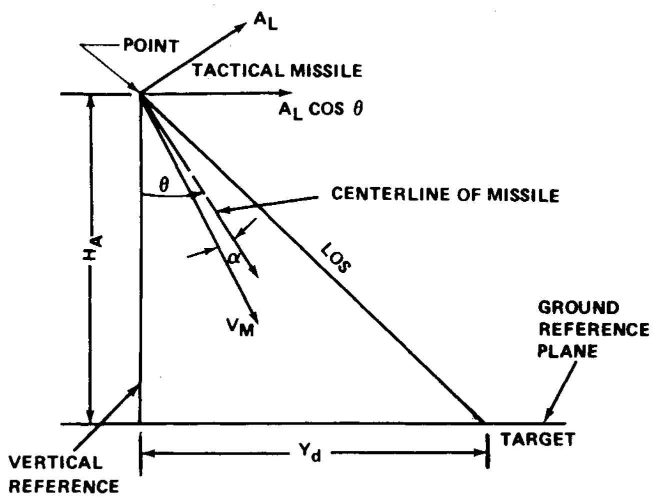


Fig. 5 Geometry of tactical missile target position.

$$x = (Y_d, \dot{Y}_d, A_L, \theta)^T = (Y_t - Y_m, \dot{Y}_t - \dot{Y}_m, A_L, \theta)^T \tag{10}$$

where Y_d is the position variable from the missile to the target projected on the ground reference frame, Y_t the position variable of the target, Y_m the position variable of the missile projected on the ground reference frame, $\dot{Y}_d$ the derivative of Y_d, A_L the lateral acceleration of the missile, θ the body attitude angle of the missile, and α the angle of attack of the missile shown in Fig. 5.

This optimal control problem will yield a controller of the form

$$u = C_Y Y_d + C_{\dot{Y}} \dot{Y}_d + C_\theta \theta + C_{A_L} AL \tag{11}$$

where C_Y, $C_{\dot{Y}}$, C_θ, and C_{A_L} are time-varying coefficients chosen to minimize the cost functional

$$J = Y_d{}^2(t_f) + \gamma\theta^2(t_f) + \beta\int_0^{t_f} u^2(t)\,\mathrm{d}t \tag{12}$$

where α and β are weighting indices.

For the case where the angle of attack cannot properly be ignored, e.g., for the larger tactical missile, the system of equations should include the angle of attack α. In addition, because it is feasible to achieve only a small angle of attack at impact, a reasonable performance index to be minimized could be

$$J = C_1 Y_d^2(t_f) + C_2\theta^2(t_f) + C_3\alpha^2(t_f) + C_4\int_{t_0}^{t_f} u^2(t)\,\mathrm{d}t \tag{13}$$

where C_n, $n = 1,4$, are weighting indices.

The performance obtainable from the optimal guidance law formulated as Eq. (13) was simulated and found to be even better than the PNG law in terms of miss distance. Additionally, it had the feature of meeting a constraint on the impact angle which the PNG law could not achieve. In particular, the impact angle was shown via an all-digital 6-DOF missile simulation to be within 1 deg of the desired impact angle and within 1 ft of the desired miss distance.[73] Other researchers have corroborated results such as these when using an optimal formulation of the guidance law.

The performance of any realistic optimal control law in a missile application is dependent on the estimation of final time or, equivalently, on time-to-go. Typically, an estimate of the range between the target and missile and the rate of change of this range are obtained from radar or other ranging devices; the time-to-go estimate is then calculated. This process works quite well as long as the range and range-rate information are accurate. In many instances, however, the data are contaminated by noise either covertly, as in the case of radar jamming devices, or inadvertently by the processing electronics. This adversely affects the estimate of time-to-go and missile performance suffers due to the degeneration of the optimal controller. Pastrick and York[85] present a discussion of several aspects of the problem in the context of a realistic missile application and provide computer algorithms for its solution, as well as a closed-form result. Another more recent attempt to estimate time-to-go was made by Fiske.[82] He also addressed the possibility of obtaining this variable by an intensity ranging technique.

Several other problems are associated with the implementation of optimal guidance laws. They appear sensitive to initial conditions, as shown by York.[86] Also, there is a strong requirement to model the system accurately. Allied to this is the importance of selecting correct numerical quantities for the elements of the weighting matrices in the chosen performance index. That task still remains more of an art than a science.

Other Guidance Schemes

A significant portion of the literature on guidance laws does not fall readily within the coverage of the four previous sections. Some of this work concentrates on specialized applications of optimization theory, particularly differential games, while others represent very simple, straightforward implementations of ad hoc controllers.

Niemi[87] presented a detailed investigation of the effects of guidance constants in a rendezvous problem. His planar model consisted of low-order linear equations with what he designated "exponential guidance." The development of guidance approximation methods for a strap-down inertial system was the focus of a paper by Ito and Tushie,[88] who again assumed a planar model.

One of the earlier applications of differential game theory to short-range missile problems occurred in an early paper by Ho et al.[89] These authors showed that variational techniques could be used to solve pursuit-evasion problems, and gave an example to prove that PNG was optimal under specific conditions. In another paper, Ho[90] considered a linear stochastic version of the intercept problem in which the target motion was random. The large number of differential equations to be solved iteratively was a major stumbling block to the implementation of the scheme. Speyer[91] extended Ho's results to the case where the missile was being tracked via radar from which it also obtained its measurements. Speyer's resulting control law was in the form of rate variable feedback with an additive random component.

Stubberud[92] generated explicit guidance laws for the pitch steering of an ascent rocket. As in the linear optimal guidance schemes, a calculation of time-to-go was required for this method. A method of successive approximations similar to quasilinearization was used to construct near-optimum guidance laws in a paper by Pfeiffer.[93] Gemin[94] studied five polynomial type approximations for nominal trajectory and sensitivity matrix computations. His results are particularly useful for preprogrammed perturbation navigation schemes.

Rawling[95] developed a guidance scheme in which the virtual miss distance was continuously calculated or which became terminal miss if no further control were applied. In another paper, Rawling[96] examined the calculation of terminal homing parameters. Chang[97] applied dynamic programming methods to solve for a minimum control effort trajectory for arbitrary initial, but fixed terminal, conditions. His paper gave a complete solution to the dynamic programming problem.

In an unusual guidance scheme for an air-to-surface missile, Whiting and Tube[98] had a virtual target "flown" on linear, circular, and quadratic trajectory segments preprogrammed into the missile guidance computer. The missile then chased the virtual target to ground. Salman and Heine[99] studied the tradeoffs between interceptor performance and sensor accuracy by examining the future sets of positions attainable by the interceptor. An aperiodic sampling algorithm was proposed by Andrew[100] to ensure that guidance errors did not grow too large between updates.

Anderson[101] developed an iterative technique for the near optimum solution of a nonlinear differential game based upon successive linearizations of a two-point boundary value problem. Later, Poulter and Anderson[102] applied this scheme to an air-to-air missile guidance problem and reported much improved simulation results compared to a PNG law. However, this was done at the cost of a greatly increased computational burden.

The large computational requirements of most guidance schemes employing high-fidelity models have attracted the attention of several authors. Liu and Han[103] suggested using continued fractions to reduce the model of a high-order nonlinear rocket. Sridhar and Gupta[104] applied singular perturbation methods to an air-to-air missile in order to reduce the computational burden. They chose the missile's rotational dynamics as the fast mode, while the position and velocity translation equations were assumed to be slow modes. Their simulation results indicated an improvement over PNG.

Gupta and Sridhar[105] also proposed a guidance scheme based on reachable sets, in which a mapping of target acceleration capabilities into missile command acceleration requirements was made. The main idea was to keep the set of future target positions within that attainable by the missile. Casler[106] examined the use of auxiliary guidance commands to introduce LOS perturbations in order to minimize the variance in key guidance parameters, prior to the terminal homing phase of the trajectory. This adaptive scheme was shown specifically to perform well in updating estimates of time-to-go. Gutman[107] considered a class of simple differential games in which the pursuer has first-order, and the evader ideal, dynamics. His results were applied to the problem of optimal guidance in the neighborhood of a collision course.

Kelly's[108] theory of disturbance-utilizing control, an extension of Johnson's[109] theory of disturbance-accommodating control (DAC), was applied to the control of a homing missile. Although the uncontrolled inputs associated with control system design are usually viewed as being detrimental, disturbances such as winds, gravity, and target maneuvers were utilized effectively in accomplishing control objectives. Optimal utilization of disturbances in the planar missile intercept of a maneuvering target was shown to provide significantly better guidance performance than that obtained by conventional linear-quadratic controller that did not account for waveform structure in the disturbance. Johnson's[110] latest contribution in this area is a discrete formulation of the DAC with an application to missile guidance.

Discussion

A survey has been made of guidance laws for short-range tactical missiles, and they have been organized into five categories for convenience of discussion. To better place them in relative perspective, Fig. 6 presents the typical hardware requirements for mechanizing them. Though the concepts are reasonably self-explanatory, it should be noted that the BR and CLOS concepts are configured with aft sensor and optics while the others are forward. Also, the optimal guidance scheme suggests the need for a microprocessor computer to handle the more complex guidance algorithms.

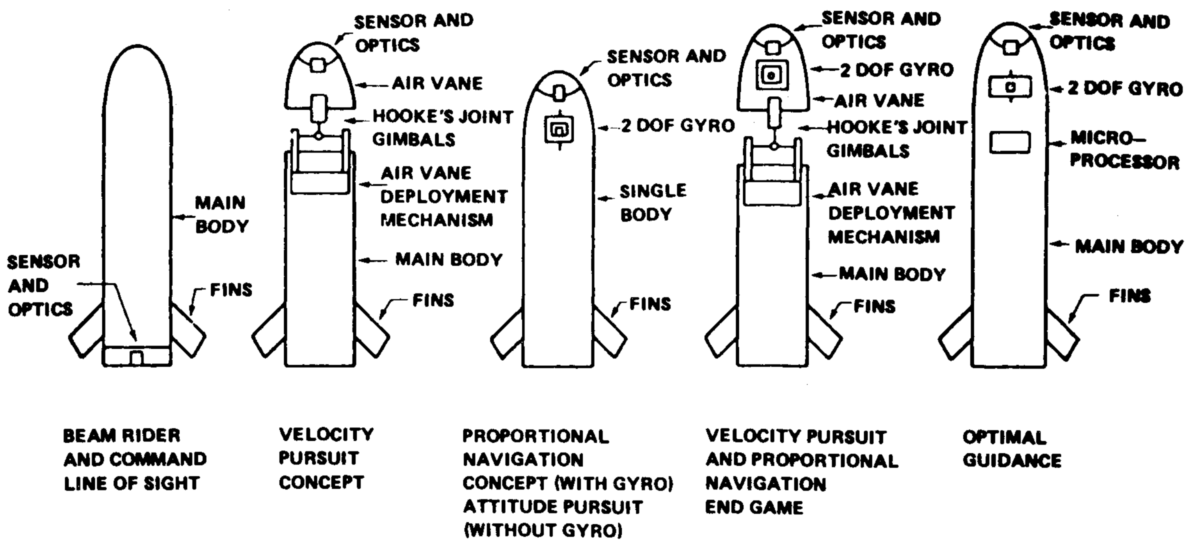


Fig. 6 Instrumentation configuration concepts.

Table 1 Guidance laws for short-range tactical missiles

	Line-of-sight		Pursuit			
	CLOS	Beamrider	Attitude	Velocity	PNG	Optimal
Ability to engage targets						
Accuracy (ft CEP)	<2	<2	>30	>20	<5	<1
Maneuverability	Low	Low	Low	Low	Const velocity	Accelerating
Additional criteria [$Y_d(t_f)$]	No	No	No	No	No	Yes
Complexity/reliability						
State required	λ_T, λ_M	λ_T, λ_M	LOS	LOS	LOS	Full
On-board gyro (ref)	Attitude	Attitude	No	No	2 DOF	2 DOF
Gimbal mechanization (seeker)	No	No	No	Air vane	Gyro	Gyro
On-board electronics	Same	Same	Same	Same	Same	Microcomputer
Cost (on-board)	0.8	0.9	1.0	1.4	1.6	1.8
Sensor requirements	Wire link	Optical link	Seeker (wide FOV)	Seeker (narrow FOV)	Seeker	Seeker (states measured and estimated)
Airframe/propulsion requirements	Low	Low	High	High	Low	High
Tactical considerations						
"Fire and forget"	No	No	Possible	Possible	Possible	Possible
Quick reaction time	No	No	Yes	Yes	Yes	Yes

Guidance concept configuration comparisons depicted in Table 1 feature relative cost and other items that show complexity and performance. With the exception of the microprocessor requirement for the optimal guidance law, all onboard electronics are comparable. However, it is important to note that the CLOS and BR concepts have vastly more complex ground stations inherent in their mechanizations. For this reason alone, their airborne complexity is relatively low and less expensive compared to the others. Their ground trackers are not included in that cost estimate.

Barring unforeseen breakthroughs in the fifth category (Other Guidance Schemes), comparison of performance characteristics favors PNG for defeating targets that are either stationary or moving with a constant velocity. PNG is simple, requires a state measurement that is easily attainable, and is relatively easy to implement. Further, if miss distance is the only criterion of performance, PNG is favorable. However, for highly maneuverable accelerating (or decelerating) targets, optimal guidance laws are superior in performance to other forms of guidance. If other performance criteria are important, such as impact angle at the target, then these criteria can be incorporated into the performance index and optimal guidance is superior in this respect.

As indicated, most linear optimal guidance laws require time-to-go to be implemented. Thus far, a direct means of measuring this important variable has not been devised, although considerable attention has been given to its estimation, e.g., the works by York[111] and Riggs.[112]

Since small size and light weight future microcomputers can be anticipated, maximum use will be made of the program flexibility offered by digital implementation of guidance laws (including state estimators or observers, if utilized) and autopilots. It is important to exploit digital implementation characteristics in this task while remaining within the constraints imposed by the digital sampling phenomena such as quantization and finite word length. The difficulties inherent in predicting and analyzing the effects of system nonlinearities are compounded with a digital system and must be addressed in the development of any advanced guidance and control system.

Summary

A literature survey was performed on guidance laws and control schemes applicable to short-range tactical missiles. Each was described briefly and discussed in terms of implementation ease, relative cost, and relative performance. The line of sight implementations require complex ground stations, even though the on-board hardware is relatively simple. Both line of sight and pursuit guidance laws have limited capability to engage maneuvering targets. Proportional navigation guidance is synthesized with relative ease and has a long history of acceptable performance and success in a large number of fielded missile weapon systems. However, it too is limited in its ability to engage targets that have a target to pursuer velocity ratio significantly greater than unity. Optimal or other control laws based on modern control theory may overcome these shortcomings. Furthermore, these formulations inherently possess the capability to have included other criteria in the specification of the cost functional in addition to miss distance in an engagement scenario. These new potential guidance laws are not without their own shortcomings. Typically they require an estimate (or measure) of the time-to-go and are sensitive to initial conditions and system model inaccuracies. Also lacking is a robust scientific approach to the selection of numerical values for the weighting matrices in the performance index. Where highly maneuverable targets or, comparably, targets with low tracking signatures may be expected, optimally derived guidance is the better solution, even with its added complexity and current implementation difficulties. However, these difficulties may be at least partially alleviated with judicious use of the microprocessor technology now enjoying a high state of development activity.

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